Propagation of Potential in Discharge Tubes

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The speed of propagation of potential waves in long discharge tubes containing dry air has been measured as a function of pressure in three tubes with internal diameters of 1.7, 5 and 18 mm. Positive and negative impulsive potentials of approximately 125 kv were used. In general, in the low pressure range the speed increases with increasing pressure. From 0.02 to 0.2 mm the curves are steep; above this range there is a decided flattening. At relatively high pressures the speed decreases and the wave shape is greatly distorted. No appreciable difference between the speeds in dry air, CO_2 or H_2, could be detected in the 5 mm tube. The speed at constant pressure in dry air is approximately a linear function of the applied voltage (75–180 kv–5 mm tube). The initial wave which traverses the tube from high voltage end to the grounded end is followed immediately by a return discharge wave starting at the grounded end. The speed of this wave is about 10^{10} cm/sec. and is apparently independent of tube diameter. There is a slight increase in speed with pressure. Maximum currents in the initial wave were measured as a function of pressure. Voltage attenuation and some knowledge of the field in the wave fronts were obtained. The speed is found to obey the principle of similarity. Speeds in the 18 and 5 mm tubes are the same at pressures in the small tube 3.6 times those in the large tube. Speeds in the 1.7 mm tube are higher than they should be according to this principle. This is probably due to the high current density (4000 amp./cm^2).

THE potential wave traversing a long discharge tube to which an impulse voltage is applied was shown in our earlier work[1] to have essentially the same characteristics as the accompanying wave of luminosity.[2] The time required for the potential wave to travel between two electrodes in the discharge tube was found to increase both with decreasing pressure and applied voltage and to be of the same order of magnitude as the time determined for the luminous wave to travel an equal distance. After this initial wave reached the grounded end of the tube a discharge wave starting at the grounded end and moving with about 1/3 the speed of light returned to the input end. Following this there were a few reflected waves rapidly decreasing in intensity superimposed upon the damped oscillation of the circuit as a whole.

The purpose of the present work was to investigate the times of travel as a function of pressure and voltage in tubes of different diameters. In addition to these measurements, the current-time curves and the values of maximum currents in the initial wave have been obtained together with some knowledge of wave fronts and voltage attenuation. Most of the measurements have been carried out with dry air although some were made with H_2 and CO_2.

[1] Snoddy, Beams and Dietrich, Phys. Rev. **50**, 469 (1936); **50**, 1094A (1936); **51**, 1008A (1937).
[2] Beams, Phys. Rev. **36**, 997 (1930).

APPARATUS AND METHOD

The apparatus is shown schematically in Fig. 1. It is essentially the same as that used in the preliminary investigation. Three Pyrex tubes (T) with internal diameters of 1.7, 5 and 18 mm were used. The lengths of tubing were joined by external glass sleeves waxed in place with Apiezon "W." Care was taken to keep all wax out of the discharge path. The terminal electrodes E_1, E_4 were brass rods with hemispherical ends and the electrodes E_2, E_3 short lengths of brass tubing. The oil pump and drying system were attached at P and the pressure read on a type of Pirani gauge at G. Pressures above the range of this gauge were read on a mercury manometer. A cold trap was placed in series with the manometer to prevent any appreciable amount of Hg vapor from diffusing into the system. The pump was shut off and the system allowed to come to equilibrium before observations were taken. As much H_2O vapor as possible was removed from the tube walls by long continued pumping and repeated discharging with a high current input. The applied potential was obtained from the Marx circuit C. A high speed, high voltage oscillograph of the Dufour type was used to determine the potential variations. The oscillograph circuit and the method used to synchronize the oscillograph and supply potential circuits are not shown as they followed standard practice for such problems. The electrodes E_2, E_3 were connected to the oscillograph deflection plates through symmetrical capacity dividers in such a manner that simultaneous equal potentials at E_2 and E_3 would produce no displacement of the electron beam. With this arrangement a

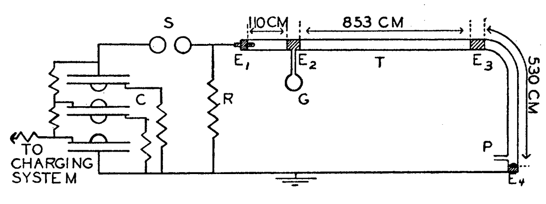

FIG. 1. Schematic diagram of apparatus showing input circuit and discharge tube.

potential wave passing down the tube produces a deflection of the beam when it arrives at E_2. This deflection is decreased when the wave arrives at E_3, the amount of decrease depending upon the voltage attenuation between E_2 and E_3. In like manner a return wave can be followed from E_3 to E_2. This oscillogram, together with a knowledge of the voltage variation at each electrode separately throughout the course of the discharge, furnishes a complete description of the traveling potential wave. Both negative and positive impulses were used.

As in the case of the luminous wave[2] the potential pulse starting at E_1 traveled relatively slowly at first. Times of travel consequently were not measured from the input electrode E_1 but from the second electrode E_2 placed about 110 cm down the tube. This distance depends upon the voltage, pressure, size of the tube and the initial degree of ionization. E_2 was placed at a point such that the wave fronts of successive discharges at constant pressure did not vary appreciably. This minimizes any effect due to a variation in the initial rate of propagation and eliminates the error involved in the time lag of the breakdown process at the entering electrode. In this work the average speed from E_1 to E_2 was about 1/3 of that from E_2 to E_3.

Current-time curves were obtained by connecting the deflection plates of the oscillograph across an electrolytic resistance of 240 ohms in series with the tube between S and E_1. Because of this resistance the voltage applied to the tube was not constant. It was found, however, that the speed of the potential wave and its form were not greatly altered provided the input voltage was adjusted to compensate approximately for the drop in the resistance. In most of the work only the maximum current in the initial wave was determined. For this the output end (E_4) of the tube was disconnected from the circuit and insulated. The current was computed from the voltage appearing across an electrolytic resistance of 40 to 80 ohms in series with the tube at the input end. This voltage was measured by a spark gap which was irradiated by ultraviolet light. While this method leads to an underestimation of the current due to the time lag of the measuring gap, the values obtained in this way are fair averages since the oscillograph showed that the current varied considerably from one discharge to the next.

RESULTS

A typical diagram of the voltage variations at the electrodes E_2 and E_3 is given in Fig. 2. In (1) the arrival of the wave at E_2 is shown at a and at b the wave has reached E_3. The time $b-c$ represents the time of transit of the initial wave from E_3 to E_4 plus the time necessary for the discharge wave to reach E_3 from the grounded end of the line. Since the gas is made highly conducting by the initial pulse, this discharge wave is similar to that obtained by grounding one end of a charged transmission line. As nearly as can be determined its speed is approximately 1/3 that of light. The part from c on represents voltage variations due to the discharge wave superimposed upon the slow damped oscillation of the circuit as a whole. At high pressures and high potentials the time $b-c$ is

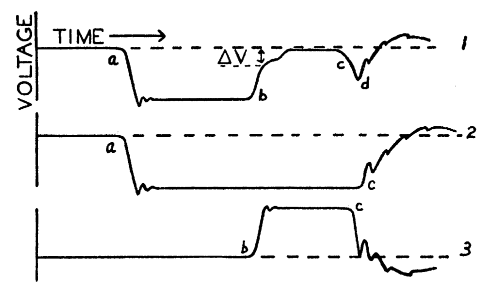

FIG. 2. General form of voltage variations observed with the oscillograph; (1) with electrodes E_2 and E_3 connected to deflection plates, (2) with E_2 alone—second deflection plate grounded, (3) E_3 alone.

variable, due at least in part to the practical difficulty of maintaining E_4 at constant potential when the gap S discharges. E_3 was placed at such a distance from E_4 that the time of transit from E_3 to E_4 and back to E_3 was always greater than the duration of the initial wave front. With this precaution the range of the measured times from a to b under conditions of constant pressure and voltage did not generally exceed 7 or 8 percent, although a few of the highest speed measurements show variations up to twice that amount. With E_4 disconnected from the circuit and insulated, the initial wave remains the same but the part from c on is lacking, showing that the last part is due to the discharge wave. ΔV represents the voltage attenuation of the wave front in traveling the distance E_2-E_3. In (2) and (3) typical voltage variations at E_2 and E_3 throughout the course of the discharge are given separately.

In Fig. 3 typical oscillograms of the voltage variations for positive and negative impulses are shown together with a current oscillogram for a positive impulse. The part $a-c$ in the current picture is characteristic of the initial wave, the part from c on shows the building up of the current after the discharge wave has returned to the input end of the tube.

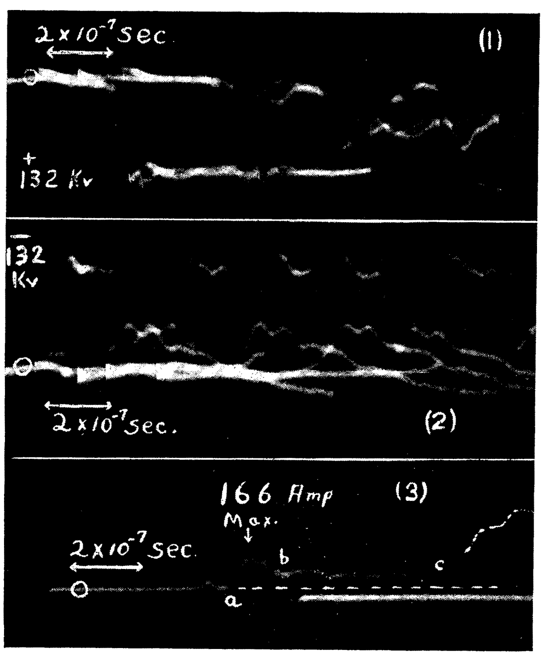

FIG. 3. Oscillograms of voltage and current. (1) Positive impulse pressure 0.18 mm 132 kv. (2) Negative impulse, pressure 0.11 mm 132 kv. (3) Voltage variation across 240 ohm resistance between S and E_1. Pressure 0.39 mm. Voltage of input condenser 160 kv.

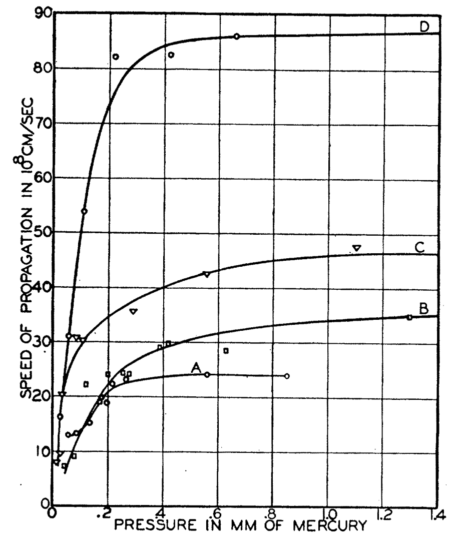

FIG. 4. Average speeds of propagation in dry air as a function of pressure. A, B, C, for positive impulse with tubes of internal diameter 1.7, 5 and 18 mm, respectively. D negative impulse 18 mm tube.

In Fig. 4 the average speeds of propagation in dry air are given as a function of pressure for tubes with inside diameter of approximately 1.7 (curve A), 5 (curve B) and 18 mm (curves C and D). The applied potential was kept constant at 125 kv for the 5 mm tube and at 132 kv for the other two tubes. Curves A, B and C are for the positive and D for the negative input voltage. The scattering of the points for a particular tube at the lower pressures is probably due to the difficulty of an exact determination of the pressure for each oscillograph trace arising from the liberation of gas from the walls of the tube by the discharges which are used to obtain proper synchronization between oscillograph and input circuits.

The negative wave is of particular interest as it probably closely resembles on a miniature scale the initial leader stroke of the lightning flash. This leader stroke travels from the cloud to ground either in a continuous or stepped manner with a maximum speed of approximately 3-5$\times 10^9$ cm/sec.[3]

[3] Schonland and Collins, Proc. Roy. Soc. **A143**, 654 (1934); Schonland, Malan and Collins, Proc. Roy. Soc. **A152**, 595 (1935); McEachron, Elect. J. **31**, 251 (1934); McEachron, Elect. World **104**, 15 (1934); Workman, Beams and Snoddy, Physics **7**, 375 (1936).

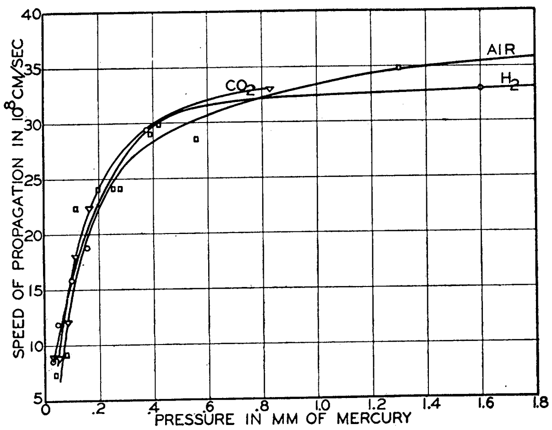

FIG. 5. Average speeds of propagation for dry air, CO_2 and H_2. Positive impulse (125 kv) in 5 mm tube.

Since the speed of the negative wave (curve D) seemed to be approaching with increase in pressure a value inconsistent with the observed values of leader velocity, one measurement was made at about 15 mm pressure. The speed had decreased to 47×10^8 cm/sec. indicating that the high value observed is probably a characteristic of the low pressure discharge, perhaps influenced in some way by the relatively small diameter tube used. In order to obtain a definite correlation with the lightning leader velocity it will be necessary to work over a much extended voltage and pressure range in a large tube. We are planning to do this in the near future.

Wave distortion and attenuation are very pronounced for the voltages used at pressures of a few mm changing entirely the character of the pulse from a wave type with a defined wave front to an approximately exponential rate of charging. As a consequence, only a few measurements were made above the range given in Fig. 4.

The speeds given here are the average speeds over the distance E_2-E_3. The time from a point halfway between E_2 and E_3 to E_3 was determined for a negative pulse (18 mm tube) at three pressures (0.021, 0.038 and 0.047 mm). The computed speeds were about 40 percent greater than the average, indicating an apparent increase in speed as the negative pulse travels down the tube. A similar determination for the positive wave (18 mm tube) over the first half of the tube gave a speed approximately 35 percent greater than the average, indicating a decrease in speed as the wave progresses. These measurements should be taken simply as an indication of the trend of the change of speed with distance down the tube for the two polarities since more work will have to be done with much longer tubes before the speed as a function of tube length can be determined with sufficient accuracy to compare absolute values.

The speed is greatly increased by the presence of appreciable ionization in the gas ahead of the wave. If two discharges are sent through the tube in rapid succession, the speed of the second pulse may be several times that of the first. Sufficient time was allowed between discharges to prevent any influence of this kind.

Figure 5 gives curves for the 5 mm tube for dry CO_2, H_2 and air with an applied voltage of 125 kv. Within the precision of measurement there is no apparent difference in speed over the pressure range used. Tank CO_2 and H_2 were used without purification other than rather careful drying over P_2O_5.

Figure 6 shows the speed as a function of positive voltage for the 5 mm tube at a constant pressure of 0.4 mm, which is above the range in which the speed varies rapidly with small changes in pressure. The relation between speed and voltage is quite accurately linear even though there was pronounced distortion and attenuation at the lower voltages.

The wave front appearing at a, Fig. 2, was not greatly influenced by changes in pressure over the range studied, although, at low pressures, it

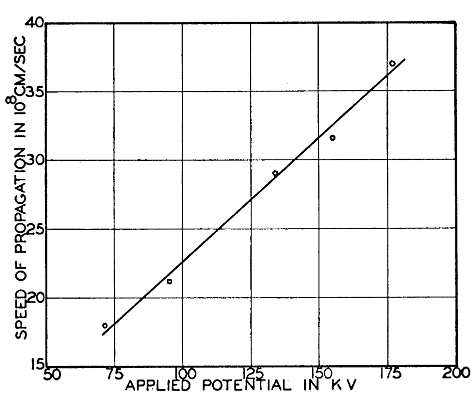

FIG. 6. Average speeds of propagation as a function of applied voltage at constant pressure of 0.4 mm. Dry air in 5 mm tube.

was measurably less steep than at high pressures. However, erratic variations due probably, in part, to distortion in the oscillograph, made a determination of the slope as a function of pressure of little value. Average values of the time from zero to 2/3 value are given in Table I.

To determine whether or not this slope was a characteristic of the input circuit rather than of the discharge, the discharge tube was replaced by a short electrolytic resistance of such a value that the maximum current through it was approximately the same as that through the tube. The rise in potential across it upon the breakdown of S was then measured. The slope of this wave was approximately the same as that at E_2 for the high pressure waves, but was definitely steeper than those at low pressure. This indicates that the wave front at E_2 is, at least in part, determined by the input circuit at high pressures but at low pressures is mainly a characteristic of the discharge tube itself.

The wave front at E_3, b of Fig. 2, and the attenuation ΔV were greatly changed as the pressure was increased. Usually, with an increase in pressure for any given voltage, the initial part of the wave front which is nearly linear became shorter and less steep, finally merging into a form represented approximately by $V(1-\epsilon^{-at})$. Values of attenuation, ΔV, are given in Table II.

TABLE I. *Average time of voltage rise at electrodes (0 to $\frac{2}{3}$ value).*

Tube Diameter mm	Time to $\frac{2}{3}$ Value	Sign of Applied Voltage
18	7.0×10^{-8} sec.	+
5	5.4	+
1.7	5.8	+
18	3.6	−

TABLE II. *Attenuation, ΔV, as a function of pressure for different tubes.*

Tube Diameter in mm	Pressure mm of Hg	ΔV Average in kv	Sign of Impulse
18	0.034	0	+
	0.56	22	
	1.1	45	
5	0.08	0	+
	0.20	25	
	0.56	40	
	1.3	60	
1.7	0.055	15	+
	0.22	25	
	0.56	31	
18	0.027	61	−
	0.055	27	
	0.22	23	
	0.42	33	
	0.66	33	

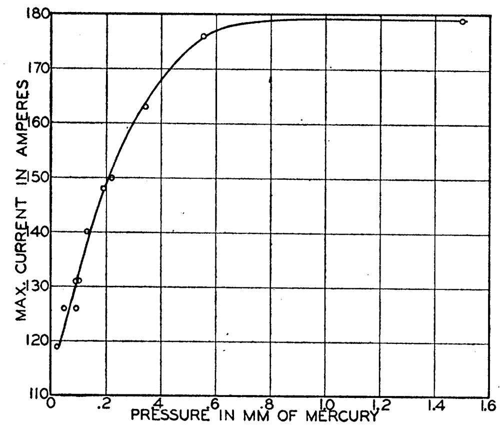

FIG. 7. Maximum current in initial wave as a function of pressure. Input condenser voltage 141 kv. Dry air in 5 mm tube.

The maximum current in the initial wave is given in Fig. 7 as a function of pressure for the 5 mm tube. The supply circuit voltage was 141 kv. For these measurements electrode E_4 was disconnected from the circuit and insulated. Curves of this type were not taken for the other tubes but the maximum current at a pressure beyond the range of rapid variation was measured. These are given in Table III.

The current-time oscillogram shown in Fig. 3 is apparently characteristic of the discharge at the higher pressures. At the lower pressures the decrease noted at b becomes more pronounced and for 0.07 mm the current at b is about 1/10 of the maximum. Beyond b it increases rapidly to a value which remains approximately constant throughout the initial wave.

The maximum voltage appearing at the output end of the tube (E_4) was determined both with the end insulated and with it connected to the return circuit through an electrolytic resistance. This voltage was measured by an irradiated spark gap. The shape of the voltage wave was also determined with the oscillograph. Curves of this type are shown in Fig. 8 for a positive impulse and 5 mm tube. Curves A, B, C give the voltage appearing across a resistance of 1.1×10^5

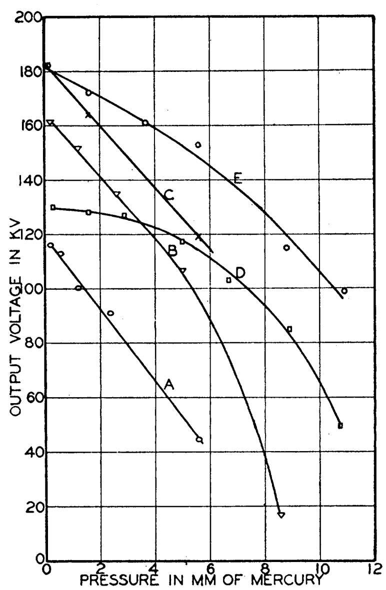

FIG. 8. Maximum voltage appearing at output end of tube (electrode E_4) as a function of pressure. A, B, C give voltages across 1.1×10^5 ohms for input voltages of 141, 180, and 194 kv respectively. For D and E, E_4 was insulated. Input voltage 141 and 194 kv, respectively. Dry air in 5 mm tube.

ohms as a function of pressure for applied voltages of 141, 180, and 194 kv respectively. For curves D and E the end of the tube was insulated ($R=\infty$). In Fig. 9 are shown several oscillograms of the voltage wave at the insulated end of the line. (Pressure 0.22 mm.)

The overvoltage at a in the oscillograms cannot be traced to any known oscillation or reflection in the circuit. It still appears when the output end of the line is shunted by a resistance as low as 30,000 ohms. At low pressures a similar although usually smaller increase can be observed in the wave appearing at the electrode E_3, even with E_4 grounded. With increasing pressure the amount of overvoltage rapidly decreases. It is apparently connected with the appearance of the minimum at b in the current-time curve (Fig. 3) and this is connected in some way with the initial building up of the propagation mechanism.

Because of this overshooting, the measured voltages in Fig. 8 at pressures below 0.4 mm are too high by amounts of approximately 3 kv for A to 8 kv for C.

The difference between the maximum voltage obtainable at E_4 at low pressure below the input voltage is about that to be expected on the basis of a sharing of charge between the capacity of the tube as a unit and the supply capacity.

If the corrected voltages for curves D and E are plotted against p^2, very good straight lines are obtained up to the pressures of about 9 mm.

The speed of the discharge wave returning from the grounded end of the tube to the input end is given in Table IV for two tubes at various pressures. As nearly as can be determined the speed is independent of the tube diameter and of the sign of the potential to which it is initially charged. It is thought, however, that the slight decrease in speed for the 18 mm tube at 0.034 mm pressure and the increase at 2.8 mm is real.

The speed of the luminous wave accompanying the potential wave was measured at a pressure of 0.1 mm and a positive voltage of 132 kv for the 18 mm tube. The method of Beams[2] which employs a high speed air turbine carrying a mirror was used. The light from two sections of the tube separated a distance of about 8 meters, after reflection from the rotating mirror was observed with a telescope fitted with a scale. The discharge in the tube supply circuit was tripped off by an auxiliary circuit which was started, when the mirror was in the proper position, by a discharge between two capillary gaps and two points extending a few mm from the rotating turbine. In this way very fair synchronization could be obtained between the mirror position and the time of discharge. For this work the output end of the tube was insulated.

These measurements were not made with any high degree of accuracy but were sufficient to show that, under the given conditions, the luminous wave and the potential wave traveled at speeds which did not differ by more than 20 percent. This of course gives no information

TABLE III. *Maximum current in initial wave for different tubes.*

Applied Voltage kv	Tube Diameter mm	Max. Current amp.	Pressure mm	amp./cm²
+132	18	230	0.6	90
+141	5	171	0.45	900
+180	5	143	0.08	750
+132	1.7	91	0.45	4000
−132	18	226	0.45	90

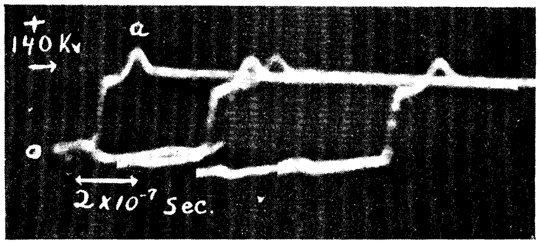

FIG. 9. Oscillograms of voltage wave at insulated end of line. Pressure (0.22 mm). 140 kv 18 mm tube.

concerning the relative positions of the potential and luminous wave fronts at any given time. A careful examination of the front of the luminous wave failed to show any characteristic shape. The wave front appeared quite straight with no apparent concentration either at the walls or toward the center of the tube. From this it would seem that the wave is not propagated as a surface discharge along the walls of the tube. It must be recognized, however, that faint luminosity $1\text{–}2\times10^{-8}$ sec. ahead of the main wave front probably would not be noticed.

Discussion of Results

For an exact quantitative explanation of the propagation of a discharge of this type it is obvious that a number of other quantities should be known. A knowledge of electron densities, speed as a function of initial degree of ionization, ion current to the tube walls, etc., should be obtained. We are now starting work with a six inch tube about 42 feet long in which we hope that these quantities can be determined with some gas such as neon or argon.

However, there are a few relationships obtainable from this work which are of some interest. While the exact mechanism of the discharge is not known, it must depend upon a transfer of potential down the tube by ionization processes of the Townsend type. In the positive wave this necessitates a supply of electrons immediately ahead of the wave. These are probably formed by photoionization in the gas by light emitted in the wave front. It is possible of course that they may in part be derived from the tube walls. In the negative wave the electron supply is obtainable from the wave front where the electron density must be quite high. It is of interest to note, for the negative case, that the maximum speed of the initial wave approaches very closely to that of the discharge wave returning through the positive column left by the initial wave (0.89×10^{10} against 1×10^{10} cm/sec.).

One of the fundamental principles of the Townsend type of discharge is the principle of similarity. It states that a decrease in all linear dimensions of the discharge apparatus by a factor k must be compensated by an increase in pressure by the same factor to maintain some characteristic of the discharge such as the breakdown potential constant. For a constant applied potential the variation of the speed of propagation is the main characteristic and the tube diameter the only dimension to be considered. By a change in diameter from 18 mm to 5 mm, i.e., a factor of 3.6, it might be expected that the speed in the two tubes would be the same at pressures in the small tube, 3.6 times those in the large tube. Table V shows how accurately

TABLE IV. *Speed of discharge waves returning from grounded end to input end of tube.*

Tube Diameter mm	Pressure mm	Speed cm/sec.	Voltage of Charged Tube
18	0.034	0.92×10^{10}	+132 kv
	0.29	1.03	
	0.56	1.00	
	1.1	1.11	
	3.9	1.07	
18	0.42	1.06×10^{10}	−132
	0.66	1.06	
	2.8	1.23	
1.7	0.055	1.07×10^{10}	+132
	0.086	1.07	
	0.18	.98	
	0.26	1.10	
	0.56	1.09	

TABLE V. *Illustration of principle of similarity.*

Pressure mm	Tube Diameter in mm	Speeds cm/sec.
0.016	18	8×10^{8}
0.0576	5	8
0.025	18	14
0.09	5	15
0.05	18	23
0.18	5	22.2
0.1	18	29.5
0.36	5	28
0.2	18	34.6
0.72	5	33.2
0.3	18	37.4
1.08	5	35

TABLE VI. *Field in wave front at electrodes E_2 and E_3.*

Pressure mm	Potential Gradient volts/cm E_2	E_3	
0.04	2300	2200	Applied voltage +125 kv
0.08	1810	2800	
0.12	870	910	
0.20	750	910	
0.42	920	740	
0.56	670	560	
1.3	420	340	

this relation holds over a considerable range of pressures for the positive impulse. This is in spite of the fact that the current density in the 5 mm tube is about 900 amp./cm^2. When this same principle is applied to the 1.7 mm tube the speed in it is about 9×10^8 cm/sec. too great at each pressure. Since the speed in this tube becomes practically independent of pressure at about 0.25 mm, the range available is not sufficiently great to determine whether or not this relation is accidental.

The discharge in the 1.7 mm tube is probably increased in speed by some additional process since the current density is about 4000 amp./cm^2. The presence of water vapor on the walls of the tube may have been a contributing factor since it is practically impossible to outgas thoroughly a tube of this diameter 50 feet in length. The independence of speed and pressure occurring at about 0.25 mm possibly means that the field in the positive column behind the wave front has become independent of pressure. This might be expected to happen in positive columns confined in very narrow tubes. In this way the energy supplied to the wave front is limited.

The speed in the 18 mm and 5 mm tubes for the positive impulse when plotted against log p can be fairly well represented by straight lines from pressures of 1.1 mm to 0.034 mm for the larger tube and from 5.4 mm to 0.11 mm for the smaller. Points below these pressures fall far off the curves. The ratio of pressures at which this relation fails is 3.2, which is nearly the ratio of tube diameters. This might be taken to mean that there is some change in the propagation mechanism when the mean free path becomes of the same order as the tube diameters.

The product of the attenuation ΔV times the maximum current I at any pressure when plotted against the speed at that pressure gives a linear relation (5 mm tube). This attenuation as measured represents the attenuation in the wave front and is not to be considered as uniformly distributed along the positive column back of the wave front. $\Delta V\times I$ is consequently proportional to the rate of energy loss in the wave front.

A rough idea of the magnitude of the field in the wave front can be obtained from the rate of potential rise at an electrode and the speed of propagation. Table VI gives these values at different pressures for the 5 mm tube and positive impulse of 125 kv. Although very erratic, the general decrease with pressure is evident. Presumably the speed would increase until the field in the wave front became independent of pressure.

By using the current-time oscillogram of Fig. 3 together with the known voltage, the total energy input was found to be 5.6 watt-seconds for the given pressure and voltage.

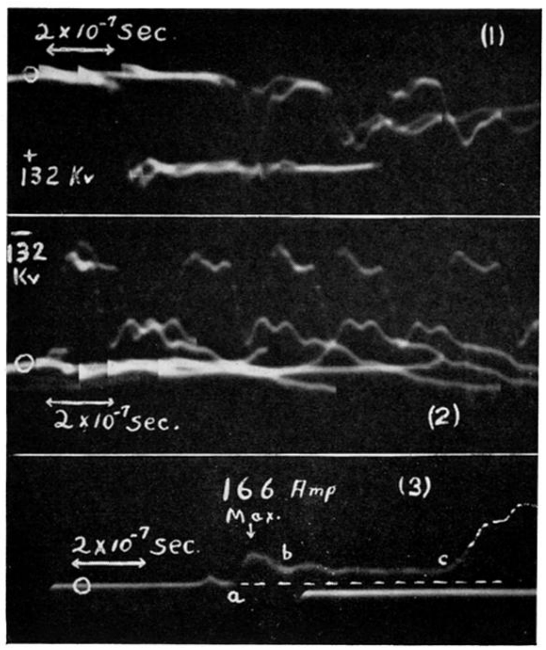

FIG. 3. Oscillograms of voltage and current. (1) Positive impulse pressure 0.18 mm 132 kv. (2) Negative impulse, pressure 0.11 mm 132 kv. (3) Voltage variation across 240 ohm resistance between S and E_1. Pressure 0.39 mm. Voltage of input condenser 160 kv.

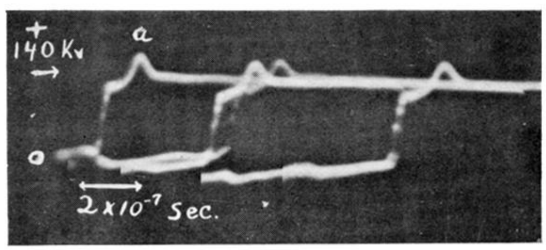

FIG. 9. Oscillograms of voltage wave at insulated end of line. Pressure (0.22 mm). 140 kv 18 mm tube.